

P-term Strings and Semi-local Strings

C. Burrage

E-mail: C.Burrage@damtp.cam.ac.uk

Department of Applied Mathematics and Theoretical Physics

Centre for Mathematical Sciences

Cambridge CB2 0WA, United Kingdom

A.C. Davis

E-mail: A.C.Davis@damtp.cam.ac.uk

Department of Applied Mathematics and Theoretical Physics

Centre for Mathematical Sciences

Cambridge CB2 0WA, United Kingdom

ABSTRACT: P-term potentials can give rise to Nielsen-Olesen or semi-local cosmic strings. We present a general analysis of these cosmic strings where we derive the Bogomol'nyi equations and field profiles for both types of string and discuss their stability. We give an analysis of the fermionic zero modes that could live on the strings and a brief discussion of the inflationary period preceding their formation.

Contents

1. Introduction	1
2. The P-term model	3
3. Field Profiles for P-term Strings	5
3.1 NO Strings	5
3.2 Semi-Local Strings	7
4. Zero Modes	10
4.1 The index theorem	10
4.2 D-term strings	12
4.3 F-term and P-term strings	13
5. Inflation	13
5.1 P-term inflation	13
5.2 The D-term Case	16
5.3 String Formation at the End of Inflation	16
6. Conclusions	17
A. Euler Angles	18
B. P-term Supergravity Potential	18

1. Introduction

There has been much interest in obtaining cosmological inflation from string theory. Generically such attempts have problems obtaining a Minkowski vacuum and stabilising the moduli fields. The KKLMMT scenario [1, 2] stabilises the volume modulus and it is therefore interesting to study the properties of this model. Typically brane inflation models give rise to lower dimensional branes, which are formed at the end of inflation, with the formation of cosmic strings being generic [3, 4]. In this paper we will study the cosmic strings that form at the end of inflation, for a review see [5, 6].

The D3/D7 version of the KKLMMT model [7, 8] gives rise to a potential for the bosonic scalars known as a P-term potential. This is an N=2 supersymmetric

potential constructed from the triplet of auxiliary fields P_i which are given a vev by a triplet of Fayet-Iliopoulos (FI) terms ξ_i . This potential has the property that if we truncate to $N=1$ supersymmetry the potential can look like a D-term potential, an F-term potential or a mix of the two depending on the direction of ξ_i . P-term models were introduced in [9] with a global $SU(2, 2|2)$ superconformal gauge theory where they correspond to a dual gauge theory of supersymmetric D3/D7 branes [10]. This is broken to $N=2$ supersymmetry by giving a vev to P_i , which corresponds to a magnetic flux triplet in the D3/D7 brane construction.

It is not straightforward to construct a P-term model in supergravity. It was thought that it was not possible to include the triplet of FI terms in $N=2$ supergravity, however in [11] we demonstrated that this could be done. In this paper we construct the potential in the more familiar language of $N=1$ supersymmetry where the second supersymmetry arises for a particular tuning of the parameters. However local $N=1$ supersymmetry with an FI term requires an R-symmetry; the superpotential must be invariant under the R-symmetry and charged under the $U(1)$ associated with the FI term [12, 13, 14, 15]. This means the charges of the scalar fields in the superpotential must be different in supergravity to their supersymmetric values which has consequences for the construction of P-term models in supersymmetry, as we will see in section 2.

This paper examines the topological defects that form in P-term potentials, in particular whether they are permitted by current cosmological constraints. Simple P-term potentials have a supersymmetric minimum with an S^1 degeneracy which means that Nielsen-Olesen (NO) cosmic strings can form by the Kibble mechanism [16], however such strings typically have too high a string tension to agree with observations. We also consider the P-term model when a second set of charged chiral multiplets are included such that there is an $SU(2)$ global symmetry between the scalar multiplets. This is known as the semi-local model. Semi-local strings are not topologically stable and as a result they do not conflict with observations. Semi-local strings appear when global and gauge symmetries coexist in a model, they were shown to arise in the developed D3/D7 brane inflation model [7], where the effective potential was P-term.

Section 2 reviews the P-term model in supersymmetry and how the charges of the fields change when we move to supergravity. In section 3 we find the general P-term form of the Bogomol'nyi equations which determine the field profiles for NO and semi-local strings. We find the asymptotic behaviour of both types of string at the core of the string and also at infinity. From this we find the string tension which can be compared with the allowed value determined by observations. In section 3.2 we also discuss the stability properties of semi-local strings which mean that they can avoid this bound.

In section 4 we discuss the zero modes that can exist on the strings. We give the supersymmetry transformations for all the fermions and then in 4.1 we discuss

the index theorem for counting fermionic zero modes in supergravity [17]. In 4.2 we discuss D-term strings which are an exceptional case because of their BPS nature, and in 4.3 we discuss F-term strings and generic P-term strings. In section 5 we discuss inflation in these potentials. Again because of their different properties we treat the generic P-term case and the D-term case in sections 5.1 and 5.2 respectively. We conclude in section 6.

2. The P-term model

We begin with a description of the P-term potential in the language of N=1 supersymmetry. Take a theory which contains three chiral multiplets with charges 0,+1,-1, and a vector multiplet and arrange the two charged scalar fields into a multiplet of charge +1

$$h = \begin{pmatrix} \phi_+ \\ \phi_-^* \end{pmatrix} \quad (2.1)$$

and define

$$P_i = h^\dagger \sigma_i h - \xi_i \quad (2.2)$$

P_i are the triplet of auxiliary fields, or moment maps, of the N=2 theory and ξ_i is a constant FI three vector [9]. The superpotential and D-term are

$$\begin{aligned} W &= \frac{\beta\phi_0}{2}(P_1 - iP_2) \\ &= \beta\phi_0 \left(\phi_+\phi_- - \frac{1}{2}(\xi_1 - i\xi_2) \right) \end{aligned} \quad (2.3)$$

$$\begin{aligned} D &= \frac{g}{2}P_3 \\ &= \frac{g}{\sqrt{2}}(|\phi_+|^2 - |\phi_-|^2 - \xi_3) \end{aligned} \quad (2.4)$$

where $\beta = g/\sqrt{2}$. Tuning the parameters in this way makes the masses of the vector and scalar particles equal, and it is in this limit that the second supersymmetry, which is anti-chiral, emerges [18]. A P-term potential in N=1 supersymmetry contains constant FI terms in the superpotential and the D-term.

The scalar potential is constructed from the superpotential and D-term as $V = |\partial W|^2 + \frac{g^2}{2}D^2$

$$V = \frac{g^2}{8} \sum_{i=1}^3 (h^\dagger \sigma_i h - \xi_i)^2 + \frac{g^2}{2} |\phi_0|^2 h^\dagger h \quad (2.5)$$

ϕ_0 is the scalar in the uncharged chiral multiplet, g is a gauge coupling constant, and σ_i are the Pauli matrices. If $\xi_1 = \xi_2 = 0$ this potential is the super-Bogomol'nyi limit¹ of a D-term potential; if $\xi_2 = \xi_3 = 0$ this is the Bogomol'nyi limit of an F-term potential. The field profiles and zero modes of F- and D-term cosmic strings in

¹as defined in [18]

supersymmetry are discussed in [19]. We split the three vector ξ_i into the product of a rotational part described by a SO(3) matrix R and a magnitudinal part $(0, 0, \xi)$

$$\xi_i = (R^{-1})_{ij} \delta_{j3} \xi \quad (2.6)$$

so all P-term potentials are described by rotations from the D-term case. The class of P-term potentials is parameterised by two Euler angles; ψ , θ , and the magnitude of the FI vector; ξ . Our conventions for Euler angles are given in appendix A.

The semi-local model includes a second pair of charged scalars such that there is an SU(2) global symmetry amongst the charged fields. The supersymmetric scalar potential becomes

$$V = \frac{g^2}{8} \sum_{i=1}^3 (h^\dagger \sigma_i h + \tilde{h}^\dagger \sigma_i \tilde{h} - \xi_i)^2 + \frac{g^2}{2} |\phi_0|^2 (h^\dagger h + \tilde{h}^\dagger \tilde{h}) \quad (2.7)$$

Semi-local strings have different stability properties to NO cosmic strings, and they therefore avoid some of the cosmological constraints that NO strings are subject to.

The potentials (2.5) and (2.7) have two types of minima; one when $|\phi_0| = 0$ and the charged fields take fixed vevs, the other when the charged fields vanish and $|\phi_0|^2 > \xi/2$. The first minimum preserves the supersymmetry the second breaks it. This is the vacuum structure required for hybrid inflation [20].

After lifting the theory to supergravity we want to be able to make comparisons with observations, hence we embed the P-term model in N=1 supergravity ignoring the second supersymmetry [10]. If a U(1) gauge theory with an FI term is to be consistently coupled to N=1 supergravity in a way that preserves gauge invariance the superpotential must be invariant under the R-symmetry but transform under the U(1) symmetry. This alters the charges of the fields appearing in the superpotential. In supersymmetry the scalar fields ϕ_\pm , ϕ_0 have charges $Q_\pm = \pm 1$, $Q_0 = 0$, and the superpotential is uncharged. In the supergravity version of the P-term model the fields have charges

$$q_i = Q_i - \rho_i \frac{\xi}{M_P^2}, \quad \sum \rho_i = 1 \quad (2.8)$$

As $M_P \rightarrow \infty$ we regain $q_i = Q_i$, indeed with a generic choice of superpotential the charges always return to their supersymmetric values in this limit. $M_P \rightarrow \infty$ is known as the rigid limit of supergravity [15] and describes supersymmetry in a curved space-time. Unless the rigid limit is being considered it no longer makes sense to combine ϕ_+ , ϕ_-^* into the multiplet (2.1). A P-term model in N=1 supergravity has the following superpotential and D-term

$$W = \frac{g\phi_0}{\sqrt{2}} \left(\phi_+ \phi_- - \frac{1}{2} (\xi_1 - i\xi_2) \right) \quad (2.9)$$

$$D = \frac{g}{\sqrt{2}}(q_0|\phi_0|^2 + q_+|\phi_+|^2 + q_-|\phi_-|^2 - \xi_3) \quad (2.10)$$

P-term potentials were constructed in [11] directly in N=2 supergravity where each component of the moment map P_i contained a constant term. Following [21] this was done by considering the truncation to N=1 supergravity where FI terms were present in the D-term and superpotential of the reduced N=1 supergravity theory. Terms involving the uncharged scalar field were found in the D-term as a result of the non-Abelian gauging of the N=2 theory needed to produce a P-term potential. This should be compared with the presence of the $q_0|\phi_0|^2$ term in (2.10).

3. Field Profiles for P-term Strings

Cosmic strings are one dimensional topological defects which form at the end of hybrid inflation for P-term potentials. We shall consider the form of NO strings, which form when there are two charged chiral multiplets in the model, and semi-local strings, which form when a second set of charged chiral fields are included. In our discussion of cosmic strings we will use the supersymmetric P-term potential as the vevs of the scalar fields are always very far from the Planck scale so the supergravity corrections to the potential are negligible.

3.1 NO Strings

We present here a unified formalism for NO strings forming in P-term potentials. The most general metric for a straight, static string [22] is

$$ds^2 = dt^2 - dz^2 - dr^2 - C^2(r)d\theta^2 \quad (3.1)$$

The string energy integral can be written as a sum of positive semidefinite terms and a surface integral term [23],

$$\begin{aligned} \mu_{\text{string}} = \int dr d\theta C \bigg(& \frac{1}{2} \left| \left(D_r + i \frac{1}{C} D_\theta \right) (Uh)_1 \right|^2 \\ & + \frac{1}{2} \left| \left(D_r + i \frac{1}{C} D_\theta \right) (Uh)_2^* \right|^2 \\ & + \frac{1}{2} \left(B - \frac{g}{2} (h^\dagger U^\dagger \sigma^3 U h - \xi) \right)^2 \\ & + \frac{g^2}{8} (h^\dagger U^\dagger \sigma^2 U h)^2 + \frac{g^2}{8} (h^\dagger U^\dagger \sigma^1 U h)^2 \\ & + \frac{g^2}{2} |\phi_0|^2 |Uh|^2 + \frac{1}{C} (\nabla \times \underline{J})_z \bigg) \end{aligned} \quad (3.2)$$

where the last term can be rewritten as a surface integral at infinity. D_μ is a co-variant derivative $D_\mu \phi_i = (\partial_\mu - igQ_i A_\mu) \phi_i$, A_μ is the gauge potential, $F_{\mu\nu}$ is the

corresponding gauge field, and $B = F_{12}$.

$$J_\mu = \frac{1}{2}i \left((D_\mu(Uh)^\dagger)(Uh) - (Uh)^\dagger D_\mu(Uh) \right) + gA_\mu \xi \quad (3.3)$$

and U is the $SU(2)$ rotation related to the $SO(3)$ rotation R , which is given in terms of Euler angles in appendix A. It is possible to relate J_μ to the gravitino potential; by choosing the trivial Kähler potential

$$K = \frac{|\phi_+|^2 + |\phi_-|^2 + |\phi_0|^2}{M_P^2} \quad (3.4)$$

the gravitino potential is $A_\mu^B = J_\mu/M_P^2$. The minimum value for μ_{string} is obtained when each of the squared terms in (3.2) vanishes, these conditions are known as the Bogomol'nyi equations. In particular they require

$$(Uh)_2 \equiv 0 \quad (3.5)$$

If the Bogomol'nyi equations are satisfied the Einstein equation becomes

$$C' + \frac{1}{M_p^2} J_\theta = \text{const} \quad (3.6)$$

The most general form of h for a straight, static, infinite cosmic string lying along the z axis is

$$h \equiv \begin{pmatrix} \phi_+ \\ \phi_-^* \end{pmatrix} = e^{in\theta} \begin{pmatrix} f(r) \\ e^{i\Delta} h(r) \end{pmatrix} \quad (3.7)$$

where $f(r)$ and $h(r)$ are real functions of the radial coordinate and Δ is a real constant. Without loss of generality we take n , the winding number of the string, to be positive. Thus (3.5) gives

$$ah(r) = be^{i\Delta} f(r) \quad (3.8)$$

where $a = e^{i(\phi+\psi)/2} \cos(\theta/2)$ and $b = ie^{i(\phi-\psi)/2} \sin(\theta/2)$ are Cayley-Klein parameters of the rotation. The most general form of the gauge potential is

$$A_r = 0, \quad A_\theta = \frac{n\alpha(r)}{g} \quad (3.9)$$

so that $B = n\alpha'(r)/gC(r)$. The boundary conditions for these fields are

$$\begin{aligned} C(0) &= 0 & C'(0) &= 1 \\ f(0) &= 0 & f(\infty) &= |a|\xi^{\frac{1}{2}} \\ \alpha(0) &= 0 & \alpha(\infty) &= 1 \end{aligned} \quad (3.10)$$

to ensure that the string energy is finite at infinity and non singular at the origin. In terms of these fields the Bogomol'nyi equations describing the minimum string energy field configuration are

$$f' - \frac{n}{C}(1 - \alpha)f = 0 \quad (3.11)$$

$$\frac{n\alpha'}{gC} = \frac{g}{2} \left(\frac{f^2}{|a|^2} - \xi \right) \quad (3.12)$$

and the Einstein equation becomes

$$1 = C' + \frac{nf^2(1-\alpha)}{M_P^2|a|^2} + \frac{n\alpha\xi}{M_P^2} \quad (3.13)$$

It is straightforward to rewrite these equations if $a = 0$. There are no known exact solution to these equations in the general case but the behaviours of the fields in the small and large r limits can be examined: As $r \rightarrow \infty$

$$f(r) \rightarrow |a|\sqrt{\xi} \quad (3.14)$$

$$\alpha(r) \rightarrow 1 \quad (3.15)$$

$$C(r) \rightarrow \left(1 - \frac{n\xi}{M_P^2} \right) r \quad (3.16)$$

and close to the origin

$$C(r) = r + \mathcal{O}(r^2) \quad (3.17)$$

$$f(r) = \beta_n r^n + \mathcal{O}(r^{n+1}) \quad (3.18)$$

$$\alpha(r) = \frac{-\xi g^2}{4n} r^2 + \mathcal{O}(r^3) \quad (3.19)$$

where β_n is a constant.

If the fields are in the Bogomol'nyi configuration we can compute the minimum energy of the string. The resulting string tension is

$$G\mu_{\text{string}} = \frac{n\xi}{4M_P^2} \quad (3.20)$$

for all P-term models. Current estimates of the string tension give $G\mu \sim 10^{-7}$ [24], which would mean that

$$\xi \sim 10^{-7} M_P^2 \quad (3.21)$$

if n is of order one. We will see in section 5 that this disagrees with the bounds on ξ coming from observations of the angular power spectrum.

3.2 Semi-Local Strings

The potential for semi-local strings was given in equation (2.7), in what follows we give a unified description of semi-local strings in P-term potentials. Repeating the same analysis as for the NO case we find that the field profiles are very similar to those of the NO string. However semi-local strings are unstable due to a degeneracy in the equations, and so unlike NO strings we would not expect any semi-local strings

formed at the end of inflation to have survived long enough to affect cosmological observations.

The string energy in the semi-local model takes the same form as in the NO case, but with the semi-local potential (2.7), and with kinetic terms for the tilded charged scalar fields. Performing the same rearrangement as in the NO case produces a set of Bogomol'nyi equations for the fields:

$$\left(D_r + i\frac{1}{C}D_\theta\right)(Uh)_1 = 0 \quad \left(D_r + i\frac{1}{C}D_\theta\right)(U\tilde{h})_1 = 0 \quad (3.22)$$

$$\left(D_r + i\frac{1}{C}D_\theta\right)(Uh)_2^* = 0 \quad \left(D_r + i\frac{1}{C}D_\theta\right)(U\tilde{h})_2^* = 0 \quad (3.23)$$

$$B - \frac{g}{2}(h^\dagger U^\dagger \sigma^3 U h + \tilde{h}^\dagger U^\dagger \sigma^3 U \tilde{h} - \xi) = 0 \quad (3.24)$$

$$(Uh)_2 = 0, (U\tilde{h})_2 = 0, |\phi_0| = 0 \quad (3.25)$$

The Einstein equation is

$$C' + A_\theta^B = 1 \quad (3.26)$$

where the constant is determined by the boundary conditions. The most general form of the bosonic profiles for a straight cosmic string satisfying (3.25) is

$$h = \frac{e^{im\theta} f_1(r)}{a^*} \begin{pmatrix} a^* \\ b^* \end{pmatrix} \quad \tilde{h} = \frac{e^{im\theta} f_2(r)}{a^*} \begin{pmatrix} a^* \\ b^* \end{pmatrix} \quad (3.27)$$

m is an arbitrary integer and without loss of generality we set $m \leq n$ so that n is the winding number of the string. The form of the gauge field is given in (3.9). Again it is straightforward to rewrite the equations if $a = 0$. The Bogomol'nyi and Einstein equations become;

$$\left(\partial_r - \frac{n}{C}(1 - \alpha)\right) f_1 = 0 \quad (3.28)$$

$$\left(\partial_r - \frac{1}{C}(m - n\alpha)\right) f_2 = 0 \quad (3.29)$$

$$\frac{n\alpha'}{gC} - \frac{g}{2} \left(\frac{f_1^2}{|a|^2} + \frac{f_2^2}{|a|^2} - \xi \right) = 0 \quad (3.30)$$

$$\frac{1}{M_P^2 |a|^2} (f_1^2 (n - n\alpha) + f_2^2 (m - n\alpha)) + \frac{n\alpha\xi}{M_P^2} = 1 - C' \quad (3.31)$$

To ensure the finiteness and regularity of the string energy the boundary conditions are

$$C(0) = 0 \quad C'(0) = 1 \quad (3.32)$$

$$f_1(0) = 0 \quad f_2(0) = f_0 \delta_{m0} \quad (3.33)$$

$$\alpha(0) = 0 \quad \alpha'(\infty) = 0 \quad (3.34)$$

$$f_1^2(\infty) + f_2^2(\infty) = |a|^2 \xi \quad (3.35)$$

where f_2 is allowed to be non-zero at the origin if $m = 0$, which is a scalar condensate at the core of the string. However $f_1(r)$ and $f_2(r)$ are not independent functions, (3.28) and (3.29) can be combined to give

$$\frac{\partial}{\partial r} \ln \left(\frac{f_2}{f_1} \right) = \frac{m - n}{C} \quad (3.36)$$

For the Bogomol'nyi equations to hold with $m \leq n$ equation (3.36) requires that as $r \rightarrow \infty$

$$f_1^2(r) \rightarrow \xi |a|^2 \quad (3.37)$$

$$f_2^2(r) \rightarrow 0 \quad (3.38)$$

and hence

$$\alpha(r) \rightarrow 1 \quad (3.39)$$

$$C(r) \rightarrow r \left(1 - \frac{n\xi}{M_P^2} \right) \quad (3.40)$$

Notice that the form of the metric at large r , and hence the deficit angle of the string, is unchanged from that of an NO string. For the fields to be finite at the origin requires $m, n \geq 0$. At small r

$$f_1(r) = \beta_n r^n + \mathcal{O}(r^{n+1}) \quad (3.41)$$

$$f_2(r) = \gamma_m r^m + \mathcal{O}(r^{m+1}) \quad (3.42)$$

$$C(r) = r + \mathcal{O}(r^2) \quad (3.43)$$

$$\alpha(r) = \frac{-\xi g^2}{4n} r^2 + \mathcal{O}(r^3) \quad (3.44)$$

If the fields are in the Bogomol'nyi configuration the string tension is

$$G\mu_{\text{string}} = \frac{\xi n}{4M_P^2} \quad (3.45)$$

which is the same as the NO case (3.20). However this does not conflict with observations as semi-local strings are unstable. As shown in [25, 26, 27] for any mode with $m < n$ there is a degeneracy in the solutions to the Bogomol'nyi equations; that is the solutions to (3.36) are a one parameter family of defects which all have the same energy. Any solution can interpolate between an NO string and a $\mathbb{C}P^1$ lump with no cost in energy. In other words the flux along the string is not confined within a tube of any particular radius and for any semi-local string generic perturbations will excite degenerate modes which will force the size of the flux tubes to ever larger values. Therefore although semi-local strings may have formed at the end of inflation, they would have been transitional and would have rapidly decayed away.

This degeneracy was invoked in [28] to produce a model of D-term inflation which did not give rise to cosmic strings. The same property means that a model of P-term inflation can be constructed which also does not conflict with observations. Infinite semi-local strings were expected to be unlikely to form in our universe for a different reason in [7]. Here it was thought that the probability of producing an infinite semi-local string from short segments would be small.

4. Zero Modes

Fermionic zero modes are a common feature of cosmic strings in supersymmetry and they alter the resulting cosmology of a model because they give rise to currents on the string. The fermions present in our system are four charged chiral fields χ_+ , χ_- , $\tilde{\chi}_+$ and $\tilde{\chi}_-$, a chargeless chiral field χ_0 , a gaugino λ and a gravitino $\psi_{\mu L}$. The supergravity transformations of the fermions are

$$\delta(\chi_{\pm}) = \frac{1}{2} \left(\sigma^r D_r + \frac{1}{C} \sigma^\theta D_\theta \right) \phi_{\pm} \bar{\epsilon} \quad (4.1)$$

$$\delta(\tilde{\chi}_{\pm}) = \frac{1}{2} \left(\sigma^r D_r + \frac{1}{C} \sigma^\theta D_\theta \right) \tilde{\phi}_{\pm} \bar{\epsilon} \quad (4.2)$$

$$\delta\chi_0 = \frac{-g}{2} ((P_1)^2 + (P_2)^2)^{\frac{1}{2}} \mathbb{1} \epsilon \quad (4.3)$$

$$\delta\lambda = i \left(\sigma_3 B + \frac{g}{2} P_3 \mathbb{1} \right) \epsilon \quad (4.4)$$

$$\delta\psi_{\mu L} = \left(\partial_\mu + \frac{1}{4} \omega_\mu^{ab} \sigma_{ab} + \frac{1}{2} i A_\mu^B \right) \epsilon_L \quad (4.5)$$

where P_μ is defined in equation (2.2). Note that if $U = \mathbb{1}$, which gives D-term strings, the system is half-BPS; half of the fermion transformations vanish.

In supersymmetry, when the gravitino is absent, it has been shown [17] that generic D-term strings have $2n$ modes of positive chirality and no modes of negative chirality. However D-term strings are normally studied away from the Bogomol'nyi limit. In the P-term model there are actually two supersymmetries present [18], the second supersymmetry is anti-chiral and gives $2n$ zero modes of negative chirality and no modes of positive chirality. So in total in the Bogomol'nyi limit where there are two supersymmetries present a D-term model has $2n$ zero modes of each chirality. This is expected as N=2 supersymmetry is not chiral. F-term strings have $2n$ modes of each chirality, and so we would expect to find $2n$ zero modes of each chirality for a P-term string in supersymmetry.

4.1 The index theorem

We will use the index theorem of [17] to calculate the number of fermionic zero modes of P-term cosmic strings in N=1 supergravity. An important consideration

is whether or not the strings are BPS, as this changes the way the index theorem is calculated because the mass matrix becomes block off diagonal. Therefore when counting the zero modes of P-term cosmic strings we have to consider the D-term case separately. In what follows we describe how to compute the number of zero modes for BPS strings as an example of how the index theorem is formulated. The general case proceeds in a similar way and for full details we refer the reader to [17].

The number of zero modes of positive and negative chirality is calculated by considering the normalisability of the fermionic zero modes at the origin and at infinity. In the BPS case the Dirac mass matrix is block off diagonal with entries in the $n_1 \times n_2$ upper right corner and $n_2 \times n_1$ lower left corner of the matrix. The fermions diagonalise the string generator such that $T_s \chi^a = q_a \chi^a$. If q_a is the charge of the a -th fermion then we write $q_a^{(1)} = q_a$ for $a = 1 \dots n_1$ and $q_a^{(2)} = q_{a+n_1}$ for $a = 1 \dots n_2$. We chose η so that $q_a^{(1)} + \eta \in \mathbb{Z} + 1/2$ and $q_a^{(2)} - \eta \in \mathbb{Z} + 1/2$. The number of massless fermions at infinity is n_z , the number of massless fermions with $a \leq n_1$ is n_{z1} and the number of massless fermions with $a > n_1$ is n_{z2} . The number of massive fermions is $2\bar{n}$ where $\bar{n} = n_1 - n_{z1} = n_2 - n_{z2}$.

Given a massless fermion with index a which is not the gravitino we define

$$\tilde{q}_{\pm}^{(1)} = \pm \frac{1}{2} - \eta \mp \left[\frac{C_1}{2} \mp \eta \right] \quad (4.6)$$

if $a > n_1$, or

$$\tilde{q}_{\pm}^{(2)} = \pm \frac{1}{2} + \eta \mp \left[\frac{C_1}{2} \pm \eta \right] \quad (4.7)$$

if $a \leq n_1$, where $[x]$ is the lowest integer which is strictly greater than x . $C_1 = 1 - n\xi$ and is related to the deficit angle of the string which is given by $\delta = 2\pi(1 - C_1)$.

After gauge fixing the gravitino field has three components, for a cosmic string background it is convenient to write them in terms of three independent Weyl fermions

$$\Sigma = \sigma^r \bar{\psi}_r + \sigma^\theta \bar{\psi}_\theta, \Psi = \sigma^r \bar{\psi}_r - \sigma^\theta \bar{\psi}_\theta, \Pi = \sigma^t \bar{\psi}_t - \sigma^z \bar{\psi}_z \quad (4.8)$$

To write the equations of motion for the gravitino in a form suitable for this analysis take

$$q_{\Psi}^{(2)} = q_{\psi} \mp 1, q_{\Sigma}^{(2)} = q_{\psi} \pm 1, q_{\Pi}^{(1)} = -q_{\psi} \pm 1 \quad (4.9)$$

where $q_{\psi} = -n\xi/2$. For a BPS configuration the gravitino is massless so we define

$$\tilde{q}_{\Psi\pm}^{(1)} = \pm \frac{1}{2} - \eta \mp \left[\frac{-C_1}{2} \mp \eta \right] \quad (4.10)$$

$$\tilde{q}_{\Sigma\pm}^{(1)} = \pm \frac{1}{2} - \eta \mp \left[\frac{3C_1}{2} \mp \eta \right] \quad (4.11)$$

By considering the field equations for the gravitino it can be seen that Π is pure gauge and decouples from the other fields so we can ignore it.

We define $\hat{q}_a^{(1)}$ to be the set of $q_a^{(1)}$ and n_{z2} copies of $\tilde{q}_\pm^{(1)}$ including $\tilde{q}_{\Psi\pm}^{(1)}$ and $\tilde{q}_{\Sigma\pm}^{(1)}$, ordered so that $\hat{q}_1^{(1)} \leq \dots \leq \hat{q}_{\bar{n}+n_z}^{(1)}$. Similarly $\hat{q}_a^{(2)}$ are the set of the $q_a^{(2)}$ and n_{z1} copies of $\tilde{q}_\pm^{(2)}$ ordered so that $\hat{q}_1^{(2)} \geq \dots \geq \hat{q}_{\bar{n}+n_z}^{(2)}$. The number of zero modes of positive and negative chirality is then given by

$$N^\pm = 2 \sum_{a=1}^{\bar{n}+n_z} [\pm \hat{q}_a^{(1)} \pm \hat{q}_a^{(2)}]_+ \quad (4.12)$$

where $[x]_+ = x$ if $x \geq 0$ and zero otherwise.

4.2 D-term strings

D-term strings are BPS states so we can apply the index theorem just described. For an NO string the index theorem says that there are $2(n-1)$ zero modes of positive chirality and no zero modes of negative chirality in supergravity [17]. It was suggested that the vanishing of two of the zero modes compared to the SUSY result could be a super Higgs effect [29].

To find the corresponding results for semi-local strings we consider positive and negative chirality modes separately. The field Σ decouples for positive chirality modes, so we need only consider the fields χ_+ , χ_- , $\tilde{\chi}_+$, $\tilde{\chi}_-$, χ_0 , λ , Φ . We have $n_1 = 4$, $n_2 = 3$, $n_{z1} = 3$ and $n_{z2} = 2$. Then

$$\begin{aligned} \hat{q}_1^{(1)} &= \tilde{q}_+^{(1)} = -1 - q_\psi, & \hat{q}_1^{(2)} &= q_{\chi_0} = -q_\psi \\ \hat{q}_2^{(1)} &= q_{\chi_-} = -q_\psi, & \hat{q}_2^{(2)} &= q_\lambda = q_\psi \\ \hat{q}_3^{(1)} &= q_{\tilde{\chi}_-} = -q_\psi, & \hat{q}_3^{(2)} &= \tilde{q}_+^{(2)} = q_\psi \\ \hat{q}_4^{(1)} &= \tilde{q}_{\Psi+}^{(1)} = -q_\psi, & \hat{q}_4^{(2)} &= \tilde{q}_+^{(2)} = q_\psi \\ \hat{q}_5^{(1)} &= q_{\tilde{\chi}_+} = m - q_\psi, & \hat{q}_5^{(2)} &= \tilde{q}_+^{(2)} = q_\psi \\ \hat{q}_6^{(1)} &= q_{\chi_+} = n - q_\psi, & \hat{q}_6^{(2)} &= q_\Psi = -1 + q_\psi \end{aligned} \quad (4.13)$$

so that

$$N^+ = 2(m + n - 1) \quad (4.14)$$

When considering the negative chirality modes Σ does not decouple. We have $n_1 = 4$, $n_2 = 4$, $n_{z1} = 3$ and $n_{z2} = 3$. Then

$$\begin{aligned} \hat{q}_1^{(1)} &= q_{\chi_-} = -q_\psi, & \hat{q}_1^{(2)} &= q_\Psi = 1 + q_\psi \\ \hat{q}_2^{(1)} &= q_{\tilde{\chi}_-} = -q_\psi, & \hat{q}_2^{(2)} &= \tilde{q}_-^{(2)} = 1 + q_\psi \\ \hat{q}_3^{(1)} &= \tilde{q}_{\Psi-}^{(1)} = -q_\psi, & \hat{q}_3^{(2)} &= \tilde{q}_-^{(2)} = 1 + q_\psi \\ \hat{q}_4^{(1)} &= \tilde{q}_-^{(1)} = -q_\psi, & \hat{q}_4^{(2)} &= \tilde{q}_-^{(2)} = 1 + q_\psi \\ \hat{q}_5^{(1)} &= \tilde{q}_{\Sigma-}^{(1)} = 1 - q_\psi, & \hat{q}_5^{(2)} &= q_{\chi_0} = -q_\psi \\ \hat{q}_6^{(1)} &= q_{\tilde{\chi}_+} = m - q_\psi, & \hat{q}_6^{(2)} &= q_\lambda = q_\psi \\ \hat{q}_7^{(1)} &= q_{\chi_+} = n - q_\psi, & \hat{q}_7^{(2)} &= q_\Sigma = -1 + q_\psi \end{aligned} \quad (4.15)$$

so that

$$N^- = 0 \tag{4.16}$$

In total there are $2(n + m - 1)$ zero modes. As only two of the modes vanish, this seems to reinforce the idea that there is a super-Higgs effect occurring.

4.3 F-term and P-term strings

For non-BPS strings the gravitino degrees of freedom Σ , and Π do not decouple. As the mass of the gravitino is non-zero the mass matrix is not block off diagonal and the index theorem becomes more complicated than that described here. In [17] it was shown that the inclusion of gravitinos in the analysis of F-term strings means that none of the global SUSY zero modes survive. Using the index theorem for generic mass matrices gives $N^+ = N^- = 2$, where the extra zero modes arise from the inclusion of the Π gravitino. The norm of this field is not positive definite which suggests that these may be gauge degrees of freedom. If we remove the modes coming from Π we get $N^+ = N^- = 0$. Exactly the same argument applies in the semi-local case.

Apart from the D-term case all P-term models have non-vanishing gravitino mass and the same argument that applies for F-term strings means that there are no zero modes for a generic P-term model. It is the BPS nature of the D-term case which means that the zero modes survive the coupling to supergravity.

5. Inflation

The P-term potential has the right vacuum structure to give hybrid inflation. In supersymmetry the potential has a supersymmetry breaking vacuum when $|\phi_+| = |\phi_-| = 0$ where $V = g^2 \xi^2 / 8$. This is a minimum if $|\phi_0|^2 > \xi / 2$. Once the critical value has been reached the fields waterfall down into the true minimum where cosmic strings can form. We first consider a generic P-term potential and then the special case when the FI terms appear only in the D-term. These two cases must be considered separately as the scalar fields have different charges in each case. We shall only consider a model containing two charged chiral multiplets, as moving to a semi-local model makes very little difference for inflation because during inflation the vevs of the charged fields are zero [28].

5.1 P-term inflation

If $\sin \theta \neq 0$ one of ξ_1, ξ_2 is non-zero and the combination $\phi_+ \phi_-$ in the superpotential (2.9) must be uncharged. We set $\rho_+ = \rho_- = 0$ in (2.8) so that

$$q_0 = \frac{-\xi \cos \theta}{M_P^2} \tag{5.1}$$

ϕ_0 is the inflaton and ϕ_+ , ϕ_- are the waterfall fields. We assume that the fields ϕ_+ and ϕ_- are always much less than the Planck mass, so neglecting terms of order $|\phi_{\pm}|^2/M_P^2$ and higher the supergravity scalar potential becomes

$$\begin{aligned}
V = & \frac{g^2}{2} e^{|\phi_0|^2/M_P^2} \left\{ |\phi_+|^2 |\phi_-|^2 + |\phi_0|^2 |\phi_-|^2 + |\phi_0|^2 |\phi_+|^2 \right. \\
& - \xi \sin \theta (e^{i\psi} \phi_+ \phi_- + e^{-i\psi} \bar{\phi}_+ \bar{\phi}_-) \\
& \left. + \xi^2 \sin^2 \theta \left(1 - \frac{|\phi_0|^2}{M_P^2} + \frac{|\phi_0|^4}{M_P^4} \right) \right\} \\
& + \frac{g^2}{2} (|\phi_+|^2 - |\phi_-|^2 - \xi \cos \theta) \\
& + \frac{g^2}{2} \xi^2 \cos^2 \theta \frac{|\phi_0|^2}{M_P^2} \left(\frac{|\phi_0|^2}{M_P^2} + 2 \right)
\end{aligned} \tag{5.2}$$

The full potential is given in appendix B. The direction $|\phi_+| = |\phi_-| = 0$ extremizes the potential in the ϕ_+ , ϕ_- directions and is a minimum if

$$e^{2|\phi_0|^2/M_P^2} (|\phi_0|^4 - 4\xi^2 \sin^2 \theta) > 4\xi^2 \cos^2 \theta \tag{5.3}$$

Inflation occurs as the fields roll along this valley. The inflationary potential is

$$V(|\phi_0|) = \frac{g^2 \xi^2}{2} \left(1 + 2 \frac{|\phi_0|^2}{M_P^2} \cos^2 \theta + \frac{|\phi_0|^4}{2M_P^4} (1 + \cos^2 \theta) \right) \tag{5.4}$$

The slow roll parameter η is

$$\eta = M_P^2 \frac{V''(|\phi_0|)}{V(|\phi_0|)} \tag{5.5}$$

$$\begin{aligned}
& = \frac{4 \cos^2 \theta + 6(1 + \cos^2 \theta) \frac{|\phi_0|^2}{M_P^2}}{1 + 2 \frac{|\phi_0|^2}{M_P^2} \cos^2 \theta + \frac{1}{2} (1 + \cos^2 \theta) \frac{|\phi_0|^4}{M_P^4}}
\end{aligned} \tag{5.6}$$

There is a period of slow roll when $|\eta| \ll 1$, as for all models of hybrid inflation $\epsilon \ll \eta$. To get slow roll requires $\cos^2 \theta < 1/4$, assuming that the inflaton is always less than the Planck scale. The potential is bounded away from the D-term case. Notice that η is always strictly positive so the spectral index $n \approx 1 + 2\eta$ is always greater than one in this model, which disagrees with observations [30].

The one-loop corrections to the potential are

$$\Delta V = \frac{\xi^2 g^4}{16\pi^2} \ln \left(\frac{|\phi_0|^2}{\Lambda} \right) \tag{5.7}$$

where Λ is a symmetry breaking scale. The slow roll equations of motion for the resulting effective potential are

$$H^2 \approx \frac{g^2 \xi^2}{6M_P^2} \tag{5.8}$$

$$3H \frac{|\dot{\phi}_0|}{M_P} = \frac{-g^2 \xi^2}{2} \left(\frac{g^2 M_P}{4\pi^2 |\phi_0|} + 4 \frac{|\phi_0|}{M_P} \cos^2 \theta + 2 \frac{|\phi_0|^3}{M_P^3} (1 + \cos^2 \theta) \right) \quad (5.9)$$

and we require $N = 60$ e-folds of inflation to agree with observations. The solutions to these equations fall into two classes depending on the relative values of g and θ .

If $(1 + \cos^2 \theta)g^2 < 8\pi^2 \cos^4 \theta$ then

$$\frac{|\phi_0|_N^2}{M_P^2} = \frac{A(B + e^{8A(1+\cos^2 \theta)N})}{B - e^{8A(1+\cos^2 \theta)N}} - \frac{\cos^2 \theta}{1 + \cos^2 \theta} \quad (5.10)$$

where

$$A^2 = \frac{8\pi^2 \cos^4 \theta - g^2(1 + \cos^2 \theta)}{8\pi^2(1 + \cos^2 \theta)^2} \quad (5.11)$$

and assuming $|\phi_0|_{\text{end}} \ll |\phi_0|_N$

$$B \approx \frac{\cos^2 \theta + (1 + \cos^2 \theta)A}{\cos^2 \theta - (1 + \cos^2 \theta)A} \quad (5.12)$$

To ensure $|\phi_0|_N^2$ is non-negative in (5.10) requires

$$8 \cos^2 \theta \left(1 - \frac{(1 + \cos^2 \theta)g^2}{16\pi^2 \cos^4 \theta} \right) N < 1 \quad (5.13)$$

which restricts θ and g in the following way

$$\cos^2 \theta < 4 \times 10^{-3} \quad (5.14)$$

$$g^2 < 4 \times 10^{-4} \quad (5.15)$$

Alternatively if $(1 + \cos^2 \theta)g^2 > 8\pi^2 \cos^4 \theta$ then

$$\frac{|\phi_0|_N^2}{M_P^2} = \frac{-\cos^2 \theta}{1 + \cos^2 \theta} + A \tan \left(4N(1 + \cos^2 \theta)A + \arctan \left(\frac{\cos^2 \theta}{A(1 + \cos^2 \theta)} \right) \right) \quad (5.16)$$

assuming $|\phi_0|_{\text{end}} \ll |\phi_0|_N$. Insisting that this is single valued gives

$$g^2 \leq \frac{\frac{\pi^4}{8N^2} + 8\pi^2 \cos^2 \theta}{1 + \cos^2 \theta} \quad (5.17)$$

so $0 \leq g^2 \leq 15.8$.

The COBE normalisation for the density perturbations at horizon crossing [31] is

$$\frac{1}{5\sqrt{3}\pi} \frac{V^{3/2}(|\phi_0|_N)}{V'(|\phi_0|_N)} \sim 1.9 \times 10^{-5} \quad (5.18)$$

If $g^2(1 + \cos^2 \theta) < 8\pi^2 \cos^4 \theta$ this requires $\xi \gtrsim 4 \times 10^{-5}$, and if $g^2(1 + \cos^2 \theta) > 8\pi^2 \cos^4 \theta$ then $\xi \gtrsim 3.4 \times 10^{-6}$. ξ is always too large to agree with the bounds on the string tension discussed in section 3.1. Hence NO strings cannot form at the end of inflation, but semi-local strings are allowed as they are not stable on cosmological timescales.

5.2 The D-term Case

If $\sin \theta = 0$ then there are no FI terms in the superpotential and the combination $\phi_+ \phi_-$ no longer has to be uncharged; $q_{\pm} \neq Q_{\pm}$. The resulting inflationary potential depends on the parameter $q_0 = -\rho_0 \xi / M_P^2$.

The tree level inflationary potential is

$$V(\phi_0) = \frac{g^2 \xi^2}{2} \left(\frac{\rho_0 |\phi_0|^2}{M_P^2} + 1 \right)^2 \quad (5.19)$$

and there exists a period of slow roll with $|\eta| \ll 1$, if $|\rho_0| < 1/4$. The loop corrections are as in (5.7), so that the slow roll equations are

$$H^2 \approx \frac{g^2 \xi^2}{6 M_P^2} \quad (5.20)$$

$$3H \frac{\dot{|\phi_0|}}{M_P} = \frac{-g^2 \xi^2}{2} \left(4\rho_0^2 \frac{|\phi_0|^3}{M_P^3} + 4\rho_0 \frac{|\phi_0|}{M_P} + \frac{g^2 M_P}{4\pi^2 |\phi_0|} \right) \quad (5.21)$$

If we make the reasonable assumption $g^2 < 4\pi^2$ these admit the same form of solution as the first case considered above. For $|\phi_0|_N^2$ to be non-negative requires $\rho_0 < 0$. $-1/4 < \rho_0 < 0$ means that the slow roll parameter

$$\eta = \frac{4\rho_0 \left(1 + \frac{3\rho_0 |\phi_0|^2}{M_P^2} \right)}{\left(1 + \frac{\rho_0 |\phi_0|^2}{M_P^2} \right)^2} \quad (5.22)$$

is always negative so the spectral index is less than one in agreement with observations [30]. For the density perturbations to be in agreement with observations requires

$$\xi \gtrsim 4.5 \times 10^{-4} \quad (5.23)$$

Which is again too high to agree with bounds on the string tension for NO strings. However, semi-local strings could form in the two doublet model, as in [28].

5.3 String Formation at the End of Inflation

Inflation ends either when the slow roll conditions are violated, or when the field is no longer rolling in a valley. When the fields leave the valley they waterfall down into the supersymmetric vacuum, where NO or semi-local strings may form depending on the choice of model.

It is well known that if NO strings form at the end of hybrid inflation the tension of the strings is typically too high to agree with observations. However in [32, 33, 34] it was shown that for F-term and D-term supersymmetry when the superpotential coupling β and coupling constant g were detuned, additional radiative corrections were considered and the fields were allowed to roll near the Planck scale there was

a region of parameter space which allowed both sufficient inflation and NO cosmic strings. It is probable that a similar analysis when applied to P-term inflation and cosmic strings would find that there was a region of parameter space where they were both permitted. However, this detuning would destroy the underlying supersymmetry. Moving to a semi-local model so that topologically unstable strings form at the end of inflation means that we can avoid the string tension constraints without detuning the couplings, thus keeping the underlying supersymmetry of the theory. In [35] the conflict between inflation and the resulting cosmic strings was avoided by detuning the couplings and by considering the warm inflation and curvaton scenarios, but again this would mean breaking the underlying symmetries of the model.

It is possible that the FI terms arise as vevs of fields which are fixed in a compactification scheme [36], in which case we do not need to alter the charges of the scalar fields. However in this case the bounds on ξ are still too large to allow NO strings to be formed at the end of inflation.

6. Conclusions

P-term potentials have the right vacuum structure to give hybrid inflation and cosmic strings, however NO cosmic strings have a tension which is too high to agree with observations of inflation. Moving to a semi-local model means that the strings that form are unstable and do not conflict with cosmological observations.

We have given the general solutions to the Bogomol'nyi equations for both NO and semi-local strings in P-term potentials and examined the behaviour of the fields at large and small distances from the string. All P-term strings have the same energy, which means that current estimates of the string tension put the same bound on the FI term in all P-term models. However this is not a problem for semi-local strings because of their instability.

D-term strings are BPS states whereas all other forms of P-term strings break all of the supersymmetries. This means that in an analysis of the string zero modes the D-term case must be treated separately from the generic P-term case. In supergravity D-term NO strings have $2(n - 1)$ zero modes and D-term semi-local strings have $2(n + m - 1)$ zero modes. When compared to the supersymmetry results of $2n$ and $2(n + m)$ respectively this seems to indicate that a super Higgs effect is occurring. For all other types of cosmic string forming in P-term potentials no zero modes survive the move from supersymmetry to supergravity.

We expect these cosmic strings to form at the end of a period of hybrid inflation. The charges of the scalar fields differ in supergravity from the supersymmetry values, which required a re-analysis of inflation in P-term potentials. It was found that the value of ξ required to give density perturbations of the right order to agree with observations is too high to agree with the string tension bound for NO strings. This makes the semi-local model preferable. In addition we note that only the D-term

model of inflation gives a spectral index which is less than one in agreement with observations.

Acknowledgements

This work was partially supported by PPARC.

A. Euler Angles

We use the following parameterisations for an $SO(3)$ rotation in terms of Cayley-Klein parameters

$$R = \begin{pmatrix} \frac{1}{2}(a^2 - b^{*2} + a^{*2} - b^2) & \frac{i}{2}(b^{*2} - a^2 + a^{*2} - b^2) & -ab - a^*b^* \\ \frac{i}{2}(a^2 + b^{*2} - a^{*2} - b^2) & \frac{1}{2}(b^{*2} + a^2 + a^{*2} + b^2) & -i(ab - a^*b^*) \\ ba^* + ab^* & i(ba^* - ab^*) & aa^* - bb^* \end{pmatrix} \quad (A.1)$$

which are defined in terms of Euler angles as

$$a = e^{i(\phi+\psi)/2} \cos \frac{\theta}{2} \quad (A.2)$$

$$b = ie^{i(\phi-\psi)/2} \sin \frac{\theta}{2} \quad (A.3)$$

The $SU(2)$ rotation associated with this $SO(3)$ rotation is

$$U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \quad (A.4)$$

B. P-term Supergravity Potential

the full supergravity potential can be calculated form

$$V = e^K \left(\left| \frac{\partial W}{\partial \phi_i} + \frac{\phi_i^* W}{M_P^2} \right|^2 - \frac{3|W|^2}{M_P^2} \right) + D^2 \quad (B.1)$$

With superpotential (2.9) and D-term (2.10) the supergravity potential for bosonic scalars is

$$\begin{aligned} V = & \frac{g^2}{2} e^K \left\{ |\phi_+ \phi_-|^2 \left(1 + \frac{|\phi_0|^4}{M_P^4} \right) + |\phi_0 \phi_-|^2 \left(1 + \frac{|\phi_+|^4}{M_P^4} \right) \right. \\ & + |\phi_0 \phi_+|^2 \left(1 + \frac{|\phi_-|^4}{M_P^4} \right) + \frac{3|\phi_0 \phi_+ \phi_-|^2}{M_P^2} \\ & \left. - \sin \theta \xi (e^{i\psi} \phi_+ \phi_- + e^{-i\psi} \bar{\phi}_+ \bar{\phi}_-) \right\} \end{aligned}$$

$$\begin{aligned}
& \times \left(1 + \frac{|\phi_0|^2}{M_P^2} + \frac{|\phi_0|^2}{M_P^4} (|\phi_0|^2 + |\phi_+|^2 + |\phi_-|^2) \right) \\
& + (\sin \theta \xi)^2 \\
& \times \left(1 - \frac{|\phi_0|^2}{M_P^2} + \frac{|\phi_0|^2}{M_P^4} (|\phi_0|^2 + |\phi_+|^2 + |\phi_-|^2) \right) \Bigg\} \\
& + \frac{g^2}{2} \left(-\frac{\xi \cos \theta}{M_P^2} |\phi_0|^2 + |\phi_+|^2 - |\phi_-|^2 - \cos \theta \xi \right)^2
\end{aligned} \tag{B.2}$$

References

- [1] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D68** (2003) 046005, [[hep-th/0301240](#)].
- [2] S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister, and S. P. Trivedi, *Towards inflation in string theory*, *JCAP* **0310** (2003) 013, [[hep-th/0308055](#)].
- [3] M. Majumdar and A.-C. Davis, *Cosmological creation of D-branes and anti-D-branes*, *JHEP* **03** (2002) 056, [[hep-th/0202148](#)].
- [4] S. Sarangi and S. H. H. Tye, *Cosmic string production towards the end of brane inflation*, *Phys. Lett.* **B536** (2002) 185–192, [[hep-th/0204074](#)].
- [5] J. Polchinski, *Introduction to cosmic F- and D-strings*, [hep-th/0412244](#).
- [6] A. C. Davis and T. W. B. Kibble, *Fundamental cosmic strings*, *Contemp. Phys.* **46** (2005) 313–322, [[hep-th/0505050](#)].
- [7] K. Dasgupta, J. P. Hsu, R. Kallosh, A. Linde, and M. Zagermann, *D3/D7 brane inflation and semilocal strings*, *JHEP* **08** (2004) 030, [[hep-th/0405247](#)].
- [8] K. Dasgupta, C. Herdeiro, S. Hirano, and R. Kallosh, *D3/D7 inflationary model and M-theory*, *Phys. Rev.* **D65** (2002) 126002, [[hep-th/0203019](#)].
- [9] R. Kallosh, *N = 2 supersymmetry and de Sitter space*, [hep-th/0109168](#).
- [10] R. Kallosh and A. Linde, *P-term, D-term and F-term inflation*, *JCAP* **0310** (2003) 008, [[hep-th/0306058](#)].
- [11] C. Burrage and A. C. Davis, *P-term potentials from 4-D supergravity*, *JHEP* **06** (2007) 086, [[arXiv:0705.1657](#)] [[hep-th](#)].
- [12] D. Freedman, *Supergravity with Axial Gauge Invariance*, *Phys. Rev.* **D15** (1977) 1173.
- [13] K. S. Stelle and P. C. West, *Relation between vector and scalar multiplets and gauge invariance in supergravity*, *Nucl. Phys.* **B145** (1978) 175.
- [14] R. Barbieri, S. Ferrara, D. V. Nanopoulos, and K. S. Stelle, *Supergravity, R invariance and spontaneous supersymmetry breaking*, *Phys. Lett.* **B113** (1982) 219.

- [15] P. Binetruy, G. Dvali, R. Kallosh, and A. Van Proeyen, *Fayet-Iliopoulos terms in supergravity and cosmology*, *Class. Quant. Grav.* **21** (2004) 3137–3170, [[hep-th/0402046](#)].
- [16] T. W. B. Kibble, *Topology of cosmic domains and strings*, *J. Phys.* **A9** (1976) 1387–1398.
- [17] P. Brax, C. van de Bruck, A. C. Davis, and S. C. Davis, *Fermionic zero modes of supergravity cosmic strings*, *JHEP* **06** (2006) 030, [[hep-th/0604198](#)].
- [18] A. Achucarro and J. Urrestilla, *F-term strings in the Bogomolnyi limit are also BPS states*, *JHEP* **08** (2004) 050, [[hep-th/0407193](#)].
- [19] S. C. Davis, A. C. Davis, and M. Trodden, *$N = 1$ supersymmetric cosmic strings*, *Phys. Lett.* **B405** (1997) 257–264, [[hep-ph/9702360](#)].
- [20] A. Linde, *Axions in inflationary cosmology*, *Phys. Lett. B* **259** (1991) 38.
- [21] A. Achucarro, A. Celi, M. Esole, J. Van den Bergh, and A. Van Proeyen, *D-term cosmic strings from $N = 2$ supergravity*, *JHEP* **01** (2006) 102, [[hep-th/0511001](#)].
- [22] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects*. Cambridge University Press, 1994.
- [23] A. Comtet and G. W. Gibbons, *Bogomolny bounds for cosmic strings*, *Nucl. Phys.* **B299** (1988) 719.
- [24] E. J. Copeland, R. C. Myers, and J. Polchinski, *Cosmic F- and D-strings*, *JHEP* **06** (2004) 013, [[hep-th/0312067](#)].
- [25] M. Hindmarsh, *Semilocal topological defects*, *Nucl. Phys.* **B392** (1993) 461–492, [[hep-ph/9206229](#)].
- [26] A. A. Penin, V. A. Rubakov, P. G. Tinyakov, and S. V. Troitsky, *What becomes of vortices in theories with flat directions*, *Phys. Lett.* **B389** (1996) 13–17, [[hep-ph/9609257](#)].
- [27] A. Achucarro and T. Vachaspati, *Semilocal and electroweak strings*, *Phys. Rept.* **327** (2000) 347–426, [[hep-ph/9904229](#)].
- [28] J. Urrestilla, A. Achucarro, and A. C. Davis, *D-term inflation without cosmic strings*, *Phys. Rev. Lett.* **92** (2004) 251302, [[hep-th/0402032](#)].
- [29] R. Jeannerot and M. Postma, *Chiral cosmic strings in supergravity*, *JHEP* **12** (2004) 043, [[hep-ph/0411260](#)].
- [30] D. N. Spergel *et al.*, *Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology*, [astro-ph/0603449](#).

- [31] D. H. Lyth and A. Riotto, *Particle physics models of inflation and the cosmological density perturbation*, *Phys. Rept.* **314** (1999) 1–146, [[hep-ph/9807278](#)].
- [32] J. Rocher and M. Sakellariadou, *Consistency of cosmic strings with cosmic microwave background measurements*, [hep-ph/0405133](#).
- [33] M. Sakellariadou and J. Rocher, *Constraining SUSY GUTs with cosmology*, [hep-ph/0406164](#).
- [34] J. Rocher and M. Sakellariadou, *Supersymmetric grand unified theories and cosmology*, *JCAP* **0503** (2005) 004, [[hep-ph/0406120](#)].
- [35] R. Jeannerot and M. Postma, *Confronting hybrid inflation in supergravity with cmb data*, *JHEP* **05** (2005) 071, [[hep-ph/0503146](#)].
- [36] A. Achúcarro, B. de Carlos, J. A. Casas, and L. Doplicher, *de Sitter vacua from uplifting D-terms in effective supergravities from realistic strings*, *JHEP* **06** (2006) 014, [[hep-th/0601190](#)].